Reg. No. : ....

D 2095 **Q.P. Code : [D 07 PMA 06]**

(For the candidates admitted from 2007 onwards)

M.Sc. DEGREE EXAMINATION, MAY 2013.

Second Year

Mathematics

MECHANICS

Time : Three hours Maximum : 100 marks

Answer any FIVE questions.

All questions carry equal marks.

(5 × 20 = 100)

1. (a) Define velocity-dependent potential and Rayleigh's dissipation function.

   (b) Derive the Lagrange's equation interms of dissipation function.

2. (a) Derive the Hamilton's principle for a non-holonomic system.

   (b) Discuss the motion of a hoop rolling down an inclined plane, without slipping.

3. (a) Derive the Lagrange's equations from Hamilton's principle.

(b) State Brachistochrome problem and obtain its solution.

4. (a) Derive the canonical equation of motion.

(b) State and prove the principle of Least Action.

5. (a) Show that the transformation $Q = \log\left(\frac{1}{q}\sin p\right)$, $P = q\cot p$ is canonical.

(b) Show that the transformation for a system of one degree of freedom

$$Q = q\cot\alpha - p\sin\alpha$$
$$P = q\sin\alpha + p\cos\alpha$$

satisfies the sympletic condition for any value of the parameter α. Find a generating function for the transformation.

6. (a) For a one dimensional system with the Hamiltonian $H = \frac{p^2}{2} - \frac{1}{2q^2}$ show that there is a constant of the motion $D = \frac{pq}{2} - Ht$.

(b) For the point transformation in a system of two degrees of freedom $Q_1 = q_1^2$, $Q_2 = q_1 + q_2$ find the most general transformation equation for P_1 and P_2 consistent with the overall transformation being canonical.

7. (a) Solve the Harmonic oscillation problem in one-dimension using the Hamilton-Jacobi's method.

   (b) Explain the notion of separation of variables in the Hamilton-Jacobi equation.

8. (a) Derive the Hamilton-Jacobi equation for Hamilton's characteristic function.

   (b) Show that the function

$$S = \frac{mw}{2}(q^2 + \alpha^2)\cot wt - mwq\alpha \csc wt$$

is a solution of the Hamilton-Jacobi equation for Hamilton's principal function for the linear harmonic oscillator with $H = \frac{1}{2m}(p^2 + m^2w^2q^2)$. Show also that this function generates a correct solution to the motion of the harmonic oscillator in time.

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D 2096 **Q.P. Code : [D 07 PMA 07]**

(For the candidates admitted from 2007 onwards)

M.Sc. DEGREE EXAMINATION, MAY 2013.

Second Year

Mathematics

OPERATIONS RESEARCH

Time : Three hours Maximum : 100 marks

Answer any FIVE questions.

Each question carries 20 marks.

(5 × 20 = 100)

1. (a) Solve by simplex method the following

Minimize $z = x_1 + 3x_2 + 3x_3$

Subject to $3x_1 - x_2 + 2x_3 \leq 7$

$2x_1 - 4x_2 \geq -12$

$-4x_1 + 3x_2 + 8x_3 \leq 10$

$x_1, x_2, x_3 \geq 0.$

(b) Solve the following LPP by artifical variable technique

Minimize $z = 4x_1 + 3x_2$

Subject to $2x_1 + x_2 \geq 10$

$-3x_1 + 2x_2 \leq 6$

$x_1 + x_2 \geq 6$

$x_1, x_2 \geq 0.$

2. (a) Using dual simplex method solve the LPP

Minimize $z = 2x_1 + x_2$

Subject to $3x_1 + x_2 \geq 3$

$4x_1 + 3x_2 \geq 6$

$x_1 + 2x_2 \geq 3$

$x_1, x_2 \leq 0.$

(b) Solve the transportation problem

	1	2	3	4	Supply
I	21	16	25	13	11
II	17	18	14	23	13
III	32	27	18	41	19
Demand	6	10	12	15	

3. (a) Solve the following assignment problem

	I	II	III	IV	V
1	11	17	8	16	20
2	9	7	12	6	15
3	13	16	15	12	16
4	21	24	17	28	26
5	14	10	12	11	13

(b) Prove that the dual of the dual is the primal.

4. (a) A project schedule has the following characteristics :

Activity :	1–2	1–3	2–4	3–4	3–5	4–9	5–6
Time : (weeks)	4	1	1	1	6	5	4
Activity :	5–7	6–8	7–8	8–10	9–10		
Time : (weeks)	8	1	2	5	7		

Draw the network and find the optimum critical path.

(b) A project consists of the following activities and three estimates

Activity	Least time (days)	Greatest time (days)	Most likely time (days)
1–2	3	15	6
2–3	2	14	5
1–4	6	30	12
2–5	2	8	5
2–6	5	17	11
3–6	3	15	6
4–7	3	27	9
5–7	1	7	4
6–7	2	8	5

(i) Draw the network

(ii) What is the probability that the project will be completed in 27 days.

5. (a) Explain in detail the basic differences between PERT and CPM.

(b) Explain about maximal flow model.

6. (a) Use revised simplex method to solve the LPP

Maximize $z = x_1 + x_2$

Subject to $3x_1 + 2x_2 \leq 6$

$x_1 + 4x_2 \leq 4$

$x_1, x_2 \geq 0.$

(b) Explain about the matrix method of defining the dual problem.

7. (a) Using simulation find the value of π.

(b) Consider the following Markov chain with two states $P = \begin{pmatrix} 0.2 & 0.8 \\ 0.6 & 0.4 \end{pmatrix}$ with $a^{(0)} = (0.7, 0.3)$. Determine $a^{(1)}, a^{(4)}$ and $a^{(8)}$.

8. (a) State the limitations and applications of simulation.

(b) Illustrate the use of Monto Carlo method for determining the value of I when $I = \int_2^5 x^3 dx$.

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D 2097 Q.P. Code : [D 07 PMA 08]

(For the candidates admitted from 2007 onwards)

M.Sc. DEGREE EXAMINATION, MAY 2013.

Second Year

Mathematics

TOPOLOGY

Time : Three hours Maximum : 100 marks

Answer any FIVE questions.

Each questions carries 20 marks.

(5 × 20 = 100)

1. (a) Let X be a set, let B be a basis for a topology I on X. Then prove that I equals the collection of all unions of elements of B.

(b) Let Y be a subspace of X, let A be a subset of Y, let $\overline{A}$ denote the closure of A in X. Then prove that the closure of A in Y equals $A \cap Y$.

2. (a) Prove that the image of a connected space under a continuous map is connected.

(b) State and prove the intermediate value theorem.

3. (a) Prove that every compact Hausdorff space is normal.

(b) State and prove the urysohn lemma.

4. (a) Prove that a subspace of a completely regular space is completely regular and product of completely regular spaces is completely regular.

(b) Prove that a metric space X is complete if every cauchy sequence in X has a convergent subsequence.

5. (a) Prove that the map $\hat{\alpha}$ is a group isomorphism.

(b) If $x_\circ \in S^{n-1}$ then prove that the inclusion $j:(S^{n-1}, x_\circ) \to (R^n - \circ, x_\circ)$ induces an isomorphism of fundamental groups.

6. (a) State and prove the maximum and minimum value theorem.

(b) State and prove the Lebesgue number lemma.

7. (a) If J is uncountable then prove that the product space R^J is not normal.

(b) State and prove the Urysohn metrization theorem.

8. State and prove the Ascoli's theorem.

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D 2098 Q.P. Code : [D 07 PMA 09]

(For the candidates admitted from 2007 onwards)

M.Sc. DEGREE EXAMINATION, MAY 2013.

Second Year

Mathematics

COMPUTER PROGRAMMING (C++ THEORY)

Time : Three hours Maximum : 100 marks

Answer any FIVE questions.

All questions carry equal marks.

$(5 \times 20 = 100)$

1. (a) What are the advantages of object oriented programming?

   (b) List out the difference between conventional programming and object oriented programming.

2. (a) Explain Enumerated data type.

   (b) Explain the Nested If Statement with suitable example.

3. (a) How are the operators classified?

   (b) Explain scope resolution operator with suitable example.

4. (a) What are punctuators and explain their purpose?

(b) Write a program to pick out the Largest among three numbers.

5. (a) What are header file? How it can be used in C++?

(b) Write a program to find the given number is odd or even.

6. (a) Write a program using function to find X! / F!

(b) Write a short notes on math Library function.

7. (a) Write a program to check whethere the two given matrix are equal or not.

(b) What is a constructor? Explain the role of constructors.

8. (a) List out the rules for overloading operators.

(b) Explain single and multiple inheritance with example.

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D 2099 Q.P. Code : [D 07 PMA 10]

(For the candidates admitted from 2007 onwards)

M.Sc. DEGREE EXAMINATION, MAY 2013.

Second Year

Mathematics

FUNCTIONAL ANALYSIS

Time : Three hours Maximum : 100 marks

Answer any FIVE questions.

All questions carry equal marks.

$(5 \times 20 = 100)$

1. (a) State and prove Uniform Boundedness Theorem.

   (b) State and prove Banach-Steinhaus theorem.

2. (a) State and prove the Schwarz inequality.

   (b) State and prove the Bessel's inequality.

3. (a) If P is a projection on H with range M and null space N then prove that $M \perp N \Leftrightarrow P$ is self-adjoint and in this case $N = M^{\perp}$.

(b) If T is normal then prove that $Mi's$ span H.

4. (a) Prove that $\sigma(x)$ is non-empty.

(b) Prove that $r(x) = \lim \| x^n \|^{1/n}$.

5. (a) Prove that the maximal ideal space m is a compact Hausdorff space.

(b) State and prove the Banach-Stone Theorem.

6. (a) If N is a normed linear space then prove that the closed with sphere S^* in N^* is a compact Hausdorff space in the weak* topology.

(b) If P is a projection on a Banach space B and if M and N are its range and null space then prove that M and N are closed linear subspaces of B such that $B = M \oplus N$.

7. (a) Let M be a closed linear sub space of a Hilbert space H, let x be a vector not in M and let d be the distance from x to M. Then prove that there exists a unique vector y_0 in M such that $\| x - y_0 \| = d$.

(b) If M is a closed linear subspace of a Hilbert space H, then prove that $H = M \oplus M^{\perp}$.

8. (a) If P is the projection on a closed linear subspace M of H then prove that M reduces an operator T iff $TP = PT$.

(b) Prove that two matrices in An are similar iff they are the matrices of a single operator on H relative to different bases.
