Reg. No. : ....

D 1728 **Q.P. Code : [D 07 PMA 06]**

(For the candidates admitted from 2007 onwards)

M.Sc. DEGREE EXAMINATION, MAY 2009.

Second Year

Mathematics

MECHANICS

Time : Three hours Maximum : 100 marks

Answer any FIVE questions.

All questions carry equal marks.

$(5 \times 20 = 100)$

1. (a) Derive Lagrange's equation of motion for a holonomic system.

(b) Discuss the motion of Atwood's machine using Lagrange equations.

2. (a) Obtain Lagrange's equations from Hamilton's principle for monogenic systems with holonomic constraints.

(b) Define canonical momentum. Show that the generalised momentum conjugate to a cyclic coordinate is conserved.

3. (a) Derive Hamilton's equations of motion using variational principle.

(b) Obtain Hamilton's canonical equations of motion.

4. (a) Define Poisson bracket of two functions. Show that all Poisson brackets are canonical invariants.

(b) Solve the simple harmonic oscillator problem in one dimension using a canonical transformation with reference to the Hamiltonian concept.

5. (a) Derive the Hamilton - Jacobi's partial differential equation in $(n+1)$ variables for Hamilton's principal function.

(b) Show that Hamilton's characteristic function generates a canonical transformation in which all the new coordinates are cyclic.

6. (a) Derive the equation :

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_j}\right) - \frac{\partial L}{\partial q_j} + \frac{\partial \mathcal{F}}{\partial \dot{q}_j} = 0$$ where L is the lagrangian and $\mathcal{F}$ is the Rayleigh's dissipation function.

(b) Using Lagrange equations, discuss the problem of finding the curve for which surface area is minimum.

7. (a) Derive Lagrange's equations of motion of nonholonomic systems.

(b) Discuss the principle of least action.

8. (a) Define Lagrange bracket and verify the canonical invariance of the Lagrange bracket.

(b) Discuss the solution of Hamilton Jacobi's equation.

Reg. No. : ....

D 604 **Q.P. Code : [D 07 PMA 08]**

(For the candidates admitted from 2007 onwards)

M.Sc. DEGREE EXAMINATION, DECEMBER 2009.

Second Year

Mathematics

TOPOLOGY

Time : Three hours Maximum : 100 marks

Answer any FIVE questions.

Each question carries 20 marks.

(5 × 20 = 100)

1. (a) Show that there exists an uncountable well ordered set, every section of which is countable.

(b) Let $\mathbf{B}$ and $\mathbf{B}'$ be bases for the topologies τ, τ' respectively. Show that τ' is finer than τ if and only if for each $x \in X$ and each basis element $B \in \mathbf{B}$ containing x, there is a basis element $B' \in \mathbf{B}'$ such that $x \in B' \subset B$.

(c) Let A be a subset of a Hausdorff space X. Show that x is a limit point of A if and only if every neighbourhood of x contains infinitely many points of A.

2. (a) Let $f : X \to Y$ where X and Y are topological spaces. Show that the following conditions are equivalent.

   (i) f is continuous

   (ii) for $A \subset X$, $f(\overline{A}) \subset \overline{f(A)}$

   (iii) inverse image of every closed set is closed.

   (b) Show that the topologies on $\mathbf{R}^n$ induced by euclidean metric and the square metric are the same as the product topology on R^n .

3. (a) Show that Cartesian product of connected spaces is connected.

   (b) Give an example of a connected space which is not path-connected. Justify your answer.

4. (a) Show that product of finitely many compact spaces is compact.

   (b) State and prove Lebesque number lemma.

5. State and prove Tietz extension theorem.

6. (a) Show that subspace of a completely regular space is completely regular and product of completely , regular spaces is completely regular.

   (b) Let X be a completely regular space and Y_1, Y_2 be two compact fications of X having the extension property. Show that there is a homoeomorphism ϕ of Y_1 onto Y_2 such that $\varphi(x) = x$ for all $x \in X$.

7. (a) Show that a metric space is compact if and only if it is complete and totally bounded.

   (b) Let X be a space and (Y, d) be a metric space. Show that for the space $\zeta(x, y)$, the compact open topology and the topology of compact convergence coincide.

8. (a) Show that the map $p : R \to S'$ given by $p(x) = (\cos 2\pi x, \sin 2\pi x)$ is a covering space.

   (b) Show that the fundamental group of the circle is infinite cyclic.

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   (i) f is continuous

   (ii) for $A \subset X$, $f(\overline{A}) \subset \overline{f(A)}$

   (iii) inverse image of every closed set is closed.

   (b) Show that the topologies on $\mathbf{R}^n$ induced by euclidean metric and the square metric are the same as the product topology on R^n.

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   (b) Give an example of a connected space which is not path-connected. Justify your answer.

4. (a) Show that product of finitely many compact spaces is compact.

   (b) State and prove Lebesque number lemma.

5. State and prove Tietz extension theorem.

6. (a) Show that subspace of a completely regular space is completely regular and product of completely , regular spaces is completely regular.

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   (b) Let X be a space and (Y, d) be a metric space. Show that for the space $\zeta(x,y)$, the compact open topology and the topology of compact convergence coincide.

8. (a) Show that the map $p: R \to S'$ given by $p(x) = (\cos 2\pi x, \sin 2\pi x)$ is a covering space.

   (b) Show that the fundamental group of the circle is infinite cyclic.

Reg. No. : ....

D 605 **Q.P. Code : [D 07 PMA 09]**

(For the candidates admitted from 2007 onwards)

M.Sc. DEGREE EXAMINATION, DECEMBER 2009.

Second Year

Mathematics

COMPUTER PROGRAMMING (C++ THEORY)

Time : Three hours Maximum : 100 marks

Answer any FIVE questions.

(5 × 20 = 100)

1. (a) Write any four features of OOP. Explain data abstraction and encapsulation.

   (b) Explain objects and message communication.

2. (a) Explain dynamic initialization of variables

   (b) Explain scope resolution operator. Illustrate it by means of a program.

3. (a) What is the use of new operator? Write its syntax. Give examples.

   (b) What are manipulators? Explain it by means of examples.

4. (a) Explain call by reference and return by reference.

   (b) Write a program to illustrate the formatting of the output values using both manipulators and ios function.

5. (a) Write a program to find the mean of two numbers using friend function.

   (b) Explain parameterized constructors and destructors.

6. (a) How a constructor is declared and defined? Write any three characteristics of constructor function.

   (b) Write a program to add the time in hour and minutes using objects as arguments.

7. (a) Write the rules for overloading operators.

   (b) Write a program to establish the overloading of unary minus operator.

8. (a) Write a program to overload + operator to perform addition of two complex objects.

   (b) Explain multiple inheritance and hybrid inheritance.

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Reg. No. : ....

D 606 **Q.P. Code : [D 07 PMA 10]**

(For the candidates admitted from 2007 onwards)

M.Sc. DEGREE EXAMINATION, DECEMBER 2009.

Second Year

Mathematics

FUNCTIONAL ANALYSIS

Time : Three hours Maximum : 100 marks

Answer any FIVE questions.

All questions carry equal marks.

$(5 \times 20 = 100)$

1. (a) State and prove the Hahn – Banach theorem.

   (b) Establish the closed graph theorem.

2. (a) If M is a proper closed linear subspace of a Hilbert space H, prove that there exists a non-zero vector z_0 in H such that $z_0 \perp M$.

(b) Prove that adjoint operation $T \to T^*$ on $B(H)$ has the following properties :

(i) $(T_1 + T_2)^* = T_1^* + T_2^*$

(ii) $(T_1 T_2)^* = T_2^* T_1^*$

(iii) $\|T^* T\| = \|T\|^2$.

3. (a) Define the projection P on a Banach space.

If P is the projection on a closed linear subspace M of H, prove that M is invariant under an operator T implies $TP = PTP$ and conversely.

(b) If T is normal, prove that the M_i's are pairwise orthogonal.

4. (a) Define a Banach algebra. Prove that the set of regular elements G is an open set and set of singular elements S is a closed set.

(b) Define a regular element in Banach algebra A.

(i) If r is an element of R, prove that $1 - r$ is regular.

(ii) If $1 - xr$ is regular, prove that $1 - rx$ is also regular.

5. (a) Prove that $M \to f_M$ is a one-to-one mapping of the set 'm' of all maximal ideals in A onto the set of all its multiplicate functionals.

(b) Discuss the Gelfand – Neumark theorem.

6. (a) If N and N' are normed linear spaces, then prove that the set $B(N, N')$ of all continuous linear transformations of N into N' is itself a normed linear space with respect to the pointwise linear operations and the norm defined. If N' is a Banach space, prove $B(N, N')$ is also a Banach space.

(b) State and prove Schwarz inequality also prove that

$$\|x + y\|^2 + \|x - y\|^2 = 2\|x\|^2 + 2\|y\|^2.$$

7. (a) Define self-adjoint operator on H. If T is an operator on H for which $(Tx, x) = 0$ for all x, prove that $T = 0$.

(b) If T is an operator on H, prove that the following conditions are equivalent to one another :

(i) $T^* T = I$;

(ii) $(Tx, Ty) = (x, y)$ for all x and y ;

(iii) $\|Tx\| = \|x\|$ for all x.

8. State and prove the spectral theorem.

Reg. No. : ....

D 1773 **Q.P. Code : [D 07 PMA 07]**

(For the candidates admitted from 2007 onwards)

M.Sc. DEGREE EXAMINATION, MAY 2009.

Second Year

Mathematics

OPERATIONS RESEARCH

Time : Three hours Maximum : 100 marks

Answer any FIVE questions.

Each question carries 20 marks.

(5 × 20 = 100)

1. (a) Use simplex method to solve the following linear programming problem :

Maximize $z = x_1 - x_2 + 3x_3$

subject to the constraints :

$x_1 + x_2 + x_3 \le 10$

$2x_1 - x_3 \le 2$

$2x_1 - 2x_2 + 3x_3 \le 0$ and

$x_1, x_2, x_3 \ge 0$

(b) Using graphical method, find the maximum value of

$z = 2x_1 + x_2$

subject to the constraints :

$x_1 + 2x_2 \le 10$

$x_1 + x_2 \le 6$

$x_1 - x_2 \le 2$

$x_1 - 2x_2 \ge 10$ and

$x_1, x_2 \ge 0.$

2. (a) Solve the following linear programming problem by using the two-phase simplex method. (12)

Minimize $z = x_1 + x_2$

subject to the constraints :

$2x_1 + 4x_2 \ge 4$

$x_1 - 7x_2 \ge 7$ and

$x_1, x_2 \ge 0.$

(b) Solve the following linear programming problem by Big M-method. (8)

Maximize $z = 3x_1 + 2x_2 + 3x_3$

subject to the constraints :

$2x_1 + x_2 + x_3 \le 2$

$3x_1 + 4x_2 + 2x_3 \ge 0$ and

$x_1, x_2, x_3 \ge 0.$

3. (a) Obtain the dual problem of the following primal problem :

Minimize $z = x_1 - 3x_2 - 2x_3$

subject to the constraints :

$3x_1 - x_2 + 2x_3 \le 7$

$2x_1 - 4x_2 \ge 12$

$-4x_1 + 3x_2 + 8x_3 = 10$

$x_1, x_2 \ge 0$

and x_3 is unrestricted. (8)

(b) Solve the following linear programming problem by dual simplex method. (12)

Minimize $z = x_1 + 2x_2 + 3x_3$

subject to the constraints :

$x_1 - x_2 + x_3 \ge 4$

$x_1 + x_2 + 2x_3 \le 8$

$x_2 - x_3 \ge 2$ and

$x_1, x_2, x_3 \ge 0$

4. (a) Obtain an initial basic feasible solution to the following transportation problem by Vogel's approximation method. Also obtain the optimum solution.

	D_1	D_2	D_3	D_4	Supply
S_1	3	7	6	4	5
S_2	2	4	3	2	2
S_3	4	3	8	5	3
Demand	3	3	2	2	

(b) Using graphical method, find the maximum value of

$$z = 2x_1 + x_2$$

subject to the constraints :

$$x_1 + 2x_2 \le 10$$
$$x_1 + x_2 \le 6$$
$$x_1 - x_2 \le 2$$
$$x_1 - 2x_2 \ge 10 \text{ and}$$
$$x_1, x_2 \ge 0.$$

2. (a) Solve the following linear programming problem by using the two-phase simplex method. (12)

Minimize $z = x_1 + x_2$

subject to the constraints :

$$2x_1 + 4x_2 \ge 4$$
$$x_1 - 7x_2 \ge 7 \text{ and}$$
$$x_1, x_2 \ge 0.$$

(b) Solve the following linear programming problem by Big M-method. (8)

Maximize $z = 3x_1 + 2x_2 + 3x_3$

subject to the constraints :

$$2x_1 + x_2 + x_3 \le 2$$
$$3x_1 + 4x_2 + 2x_3 \ge 0 \text{ and}$$
$$x_1, x_2, x_3 \ge 0.$$

3. (a) Obtain the dual problem of the following primal problem :

Minimize $z = x_1 - 3x_2 - 2x_3$

subject to the constraints :

$$3x_1 - x_2 + 2x_3 \le 7$$
$$2x_1 - 4x_2 \ge 12$$
$$-4x_1 + 3x_2 + 8x_3 = 10$$
$$x_1, x_2 \ge 0$$

and x_3 is unrestricted. (8)

(b) Solve the following linear programming problem by dual simplex method. (12)

Minimize $z = x_1 + 2x_2 + 3x_3$

subject to the constraints :

$$x_1 - x_2 + x_3 \ge 4$$
$$x_1 + x_2 + 2x_3 \le 8$$
$$x_2 - x_3 \ge 2 \text{ and}$$
$$x_1, x_2, x_3 \ge 0$$

4. (a) Obtain an initial basic feasible solution to the following transportation problem by Vogel's approximation method. Also obtain the optimum solution.

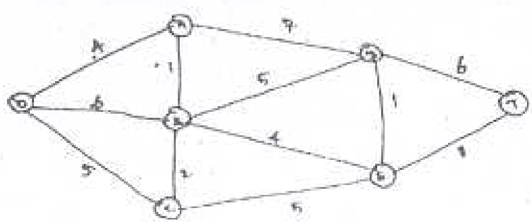
	D_1	D_2	D_3	D_4	Supply
S_1	3	7	6	4	5
S_2	2	4	3	2	2
S_3	4	3	8	5	3
Demand	3	3	2	2	

(b) A departmental head has four subordinates, and four tasks to be performed. The subordinates differ in efficiency, and the tasks differ in their intrinsic difficulty. His estimate, of the time each man would take to perform each task, is given in the matrix below :

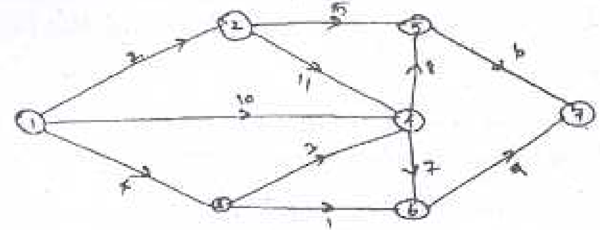
	Men			
Tasks	E	F	G	H
A	18	26	17	11
B	13	28	14	26
C	38	19	18	15
D	19	26	24	10

How should tasks be allocated, one to a man, so as to minimize the total man-hours?

5. (a) Consider the following network where the number on links represent actual distance between the corresponding nodes. Find the minimal spanning tree. (10)



(b) The network below gives the permissible routes and their lengths in miles between stations of city 1 (node 1) and six other cities (nodes 2-7).



Determine the shortest route and hence the shortest distance from city 1 to city 7. (10)

6. (a) A project schedule has the following characteristics

Activity :	(1, 2)	(1, 3)	(2, 4)	(3, 4)	(3, 5)	(4, 9)
Time :	4	1	1	1	6	5
Activity :	(5, 6)	(5, 7)	(6, 8)	(7, 8)	(8, 10)	(9, 10)
Time :	4	8	1	2	5	7

Construct the network; compute EST and LET; Also find the critical path and total project duration. (10)

(b) Consider the network shown below :