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Reg. No. : ....

D 2106 Q.P. Code : [D 07 PMA 01]

(For the candidates admitted from 2007 onwards)

M.Sc. DEGREE EXAMINATION, MAY 2014.

First Year

Mathematics

ALGEBRA

Time : Three hours Maximum : 100 marks

Answer any FIVE questions.

($5 \times 20 = 100$)

1. (a) Prove that if $o(G) = p^n$ where p is a prime number, then $Z(G) \neq e$. (5)

(b) If p is a prime number and $p^\alpha / o(G)$, show that G has a subgroup of order p^α. (5)

(c) Let G be a group and suppose that G is internal direct product of $N_1, N_2, \ldots N_n$ and let $T = N_1 \times N_2 \times \cdots \times N_n$. Show that G and T are isomorphic. (10)

2. (a) Prove that the ideal $A=(a_0)$ is a maximal ideal of the Euclidean ring R if and only if a_0 is a prime element of R. (10)

(b) Let $f(x)=a_0+a_1x+a_2x^2+\cdots+a_nx^n$ be a polynomial with integer coefficients. Suppose that for some prime number p, $p\nmid a_n\ p|a_1\ p|a_2\ \ldots\ p|a_6\ p^2\nmid a_0$. Prove that $f(x)$ is irreducible over the rationals. (10)

3. (a) Let R be an Euclidean ring and let A be an ideal of R. Show that there exists an element $a_0 \in A$, such that A consists exactly of all a_0x as x ranges over R. (6)

(b) Prove that if $f(x)$ and $g(x)\neq 0$ in $F[x]$ are two polynomials, then there exists two polynomials $t(x)$ and $r(x)$ in $F[x]$, such that $f(x)=t(x)g(x)+r(x)$ where $r(x)=0$ or $\deg r(x)<\deg g(x)$. (7)

(c) Prove that if $f(x)$ and $g(x)$ are primitive polynomials, then $f(x)g(x)$ is a primitive polynomials. (7)

4. (a) A polynomial of degree n over a field can have atmost n roots in any extension field. (10)

(b) Prove that if L is an algebraic extension of K and if K is an algebraic extension of F then L is an algebraic extension of F. (5)

(c) If $p(x) \in F[x]$ and if K is an extension of F, prove that for any element $b \in K$, $p(x) = (x-b)q(x) + p(b)$ where $q(x) \in K[x]$ and where $\deg q(x) = \deg p(x) - 1$. (5)

5. (a) Prove that if $p(x)$ a polynomial in $F[x]$ of degree $n \geq 1$ and is irreducible over F, then there is an extension E of F such that $[E:F] = n$ in which $p(x)$ has a root. (10)

(b) Show that the polynomials $f(x) \in F[x]$ has a multiple root if and only if $f(x)$ and $f'(x)$ have a non trivial common factor. (10)

6. (a) If $p(x) \in F[x]$ is solvable by radicals over F, show that the Galois group over F of $p(x)$ is a solvable group. (8)

(b) Prove that for every prime number p and every positive integer m there exists a field having p^m elements. (6)

(c) Show that the multiplicative group of non-zero elements of a finite field is cyclic. (6)

7. (a) If V is n-dimensional over F and if $T \in A(V)$ has all its characteristic roots in F, prove that T satisfies a polynomial of degree n over F. (10)

(b) Show that there exists a subspace W of V invariant under T, such that $V = V_1 \oplus W$. (10)

8. (a) If F is a field of characteristic 0 and if $T \in A_F(V)$ is such that $\operatorname{tr} T^i = 0$ for all $i \geq 1$, prove that T is nilpotent. (6)

(b) Show that the linear transformation T on V is unitary if and only if it takes an orthonormal basis of V into an orthonormal basis of V. (7)

(c) Prove that $T \geq 0$ if and only if $T = AA^*$ for some A. (7)

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D 2107 **Q.P. Code : [D 07 PMA 02]**

(For the candidates admitted from 2007 onwards)

M.Sc. DEGREE EXAMINATION, MAY 2014.

First Year

Mathematics

REAL ANALYSIS

Time : Three hours Maximum : 100 marks

Answer any FIVE questions.

Each question carries 20 marks.

$(5 \times 20 = 100)$

1. (a) Prove that $f \sum R(\alpha)$ on $[a,b]$ if and if for every $\sum > 0$ there exists a partition P such that $V(P,f,\alpha) - L(p,f,\alpha) < \sum$.

   (b) Suppose f is bounded on $[a,b]$, f has only finitely many points of discontinuous on $[a,b]$, and α is continuous at every point at which f is discontinuous. The prove that $f \sum R(\alpha)$.

2. (a) Suppose $f \sum R(\alpha)$ on $[a,b]$, $m \le f \le M$, ϕ is continuous on $[m,,M]$ and $h(x)=\phi(f(x))$ and $[a,b]$. Prove that $h \sum R(\alpha)$ on $[a,b]$.

(b) If γ' is continuous on $[a,b]$ prove that γ is rectifiable and $\wedge(\gamma)=\int_a^b |\gamma'(t)| dt$.

3. If f is a continuous complex function on $[a,b]$, prove that there exists a sequence of polynomials P_n such that $\lim_{n \to a} P_n(x)=f(x)$ uniforms an $[a,b]$. If f is real, show further that P_n can be taken as real.

4. (a) Suppose $f_n \to f$ uniformly on a set E is a metricspace. Let x be a limit point of E, and suppose that $\lim_{t \to x} f_n(t)=A_n \quad (n=1,2,3,...)$. The prove that $\{A_n\}$ converges and $\lim_{t \to x} f(t)=\lim_{n \to \infty} A_n$.

(b) Prove that there exists a real continuous function on the real line which is nowhere differentiable.

5. State and prove that inverse function theorem.

6. State and prove the implicit function theorem.

7. (a) Prove that the outer measure of an internal is its length.

   (b) State and prove that the bounded convergence theorem.

8. (a) State and prove the vitali's covering lemma.

   (b) State and prove the inequalities `done to minkowski and holder.

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D 2255 **Q.P. Code : [D 07 PMA 03]**

(For the candidates admitted from 2007 onwards)

M.Sc. DEGREE EXAMINATION, MAY 2014.

First Year

Mathematics

DIFFERENTIAL EQUATIONS

Time : Three hours Maximum : 100 marks

Answer any FIVE questions.

$(5 \times 20 = 100)$

1. (a) Write the system $u'' + 3v' + 4u + 5v = 6t$, $v'' - u' + 4u + v = \cos t$ in the vector matrix form.

(b) Verify that in the equation $x' = (\cos^2 t)x$, $\cos^2 t$ is periodic with period 2π but solutions are note periodic.

2. (a) Show that the set of all solutions of the system $x' = A(t)x$ on I forms n-dimensional vector space over the field of complex numbers.

(b) Determine e^{tA} and a fundamental matrix for the system $x' = Ax$ where $A = \begin{pmatrix} -1 & 2 & 3 \\ 0 & -2 & 1 \\ 0 & 3 & 0 \end{pmatrix}$.

3. (a) State and prove Cauchy-Peano theorem for system of equations.

(b) Compute the first three successive approximations for the solution of the equation $x' = \dfrac{x}{1+x^2}$, $x(0) = 1$.

4. (a) Show that the error due to the truncation at the n^{th} approximation tends to zero as $n \to \infty$.

(b) Solve the IVP $x' = x$, $x(0) = 1$ by the method of successive approximations.

5. (a) Determine the solution of $u_{tt} = c^2 u_{xx}$ $0 < x < l$, $t > 0$ with $u(x,0) = \sin\left(\dfrac{\pi x}{l}\right)$, $0 \le x \le l$, $u_t(x,0) = 0$, $0 \le x \le l$, $u(0,t) = 0$, $u(l,t) = 0$, $t \ge 0$.

(b) Solve $u_{xx} - u_{yy} = 1$, $u(x,0) = \sin y$, $u_y(x,0) = x$.

6. State and prove the Cauchy-Kowlalewsky theorem.

7. By the method of separation of variable, solve the telegraph equations $u_{tt} + au_t + bu = c^2 u_{xx}$, $0 < x < l$, $t > 0$, $u(x,0) = f(x)$, $u_t(x,0) = 0$, $u(0,t) = u(l,t) = 0$, $t > 0$.

8. (a) State and prove maximum principle theorem.

   (b) Reduce the Neumann problem to the Dirch let problem in the two dimensional case.

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D 2108 Q.P. Code : [D 07 PMA 04]

(For the candidates admitted from 2007 onwards)

M.Sc. DEGREE EXAMINATION, MAY 2014

First Year

Mathematics

NUMERICAL METHODS

Time : Three hours Maximum : 100 marks

Answer any FIVE question.

($5 \times 20 = 100$)

1. (a) Find a real root of the equation $x^3 + x^2 - 1 = 0$ by iteration method correct to 4 decimal places.

   (b) Evaluate $\sqrt{12}$ to four decimal places by Newton's–Raphson method.

2. (a) Find a quadratic factors of $x^4 - 1.1x^3 + 2.3x^2 + 0.5x + 3.3 = 0$ by Bairstow's method.

   (b) Evaluate $\int_0^1 \frac{dx}{1+x^2}$ using Trapezodial rule with $h = 0.2$. Hence determine the value of π.

3. (a) Using Gauss-Seidal method. Solve the system of equations.

$$8x - y + z - 18 = 0$$
$$2x + 5y - 2z - 3 = 0$$
$$x + y - 3z + 6 = 0$$

(b) Solve the following equations by LU decomposition method

$$2x + y + 4z = 12$$
$$8x - 3y + 2z = 0$$
$$4x + 11y - z = 33$$

4. (a) Solve the equations

$$9x - y + 2z = 9$$
$$x + 10y - 2z = 15$$
$$2x - 2y - 13z = -17$$

to three decimal places by relaxation method.

(b) Given $\frac{dy}{dx} = 3x + y/2$ and $y(0) = 1$. Find the values of $y(0.1)$ and $y(0.2)$ using Taylor series method.

5. (a) Using Runge-Kutta method, find the value of $y(1.1)$ given that $\frac{dy}{dx} = y^2 + xy$; $y(1) = 1$.

(b) Find the numerically largest eigen value using power method, given $A = \begin{pmatrix} 5 & 4 & 3 \\ 10 & 8 & 6 \\ 20 & -4 & 22 \end{pmatrix}$.

6. (a) Solving a second order equation by the shooting method. $y'' = 3t^2 + 2 - ty' - y$.

(b) Using Adam's method find $y(0.4)$ given $y' = \frac{xy}{2}$, $y(0) = 1$, $y(0.1) = 1.01$, $y(0.2) = 1.022$, $y(0.3) = 1.023$.

7. (a) Using Newton's divided difference formula, find $f(656)$ from the following table.

x:	654	658	659	661
y:	2.8156	2.8182	2.8189	2.8202

(b) Solve $\frac{d^2y}{dx^2} = y$, $y(1) = 1.175$, $y(3) = 10.018$.

8. (a) Solve $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$ subject to the conditions $u(x,0) = 0$, $u(0,t) = 0$ and $u(1,t) = t$. Compute u for $t = 1/8$ in two steps, using Crank-Nicolson formula.

(b) A banjo string is 80 cm long and weights 1.0 gram. It is stretched with a tension of 40,000 grams. At a point 20 cm from one end it is pulled 0.6 cm from the equilibrium position and then released. Find the displacement of points along the string as a function of time. How long does it take for one complete cycle of motion? From this compute the frequency with which it vibrates.

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D 2109 **Q.P. Code : [D 07 PMA 05]**

(For the candidates admitted from 2007 onwards)

M.Sc. DEGREE EXAMINATION, MAY 2014.

First Year

Mathematics

COMPLEX ANALYSIS

Time : Three hours Maximum : 100 marks

Answer any FIVE questions.

All questions carry equal marks.

$(5 \times 20 = 100)$

1. (a) If all the zeros of a polynomial $p(z)$ lie in a half plane, then prove that all zeros of the derivative $p'(z)$ lie in the same half plane.

   (b) Prove that the cross ratio (z_1, z_2, z_3, z_4) is real if and only if the four points lie on a circle or an a straight line.

2. (a) State and prove Cauchy's theorem for a circular disk.

(b) Prove that the line integral $\int_{\gamma} pdx + qdy$ defined in a region Ω depends only on the end points of γ if and only if there exists a function $u(x,y)$ in Ω with $\frac{\partial u}{\partial x} = p, \frac{\partial u}{\partial y} = q$.

3. (a) Suppose that $\phi(\zeta)$ is a continuous complex valued function on an arc γ. Then prove that the function $F_n(z) = \int_{\gamma} \frac{\phi(\zeta) d\zeta}{(\zeta - z)^n}$ is analytic in each of the regions determined by γ and $F_n'(z) = nF_{n+1}(z)$.

(b) State and prove Taylor's theorem.

4. (a) State and prove Cauchy's residue theorem.

(b) Evaluate $\int_{0}^{\infty} \frac{dx}{(1+x^2)^2}$.

5. (a) State and prove the mean-value property of Harmonic function.

(b) State and prove Schwarz's theorem.

6. (a) If $f_n(z)$ is analytic in the region Ω_n, and the sequence $\{f_n(z)\}$ converges to a limit function $f(z)$ in a region Ω uniformly on every compact subset of Ω, then prove that $f(z)$ is analytic in Ω and $f_n'(z)$ converges uniformly to $f'(z)$ an every compact subset of Ω.

(b) State and prove Mittag-Leffler's theorem.

7. (a) State and prove Riemann mapping theorem.

(b) Write a note on "mapping on a rectangle".

8. (a) Prove that the sum of the residues of an elliptic function is zero.

(b) Derive the differential equation satisfied by Weierstrass $\wp$–function.